

Exercise 8.1 (Revised) - Chapter 8 - Introduction To Trigonometry - Ncert Solutions class 10 - Maths

Updated On 11-02-2025 By Lithanya

NCERT Class 10 Maths: Chapter 8 - Introduction to Trigonometry Solutions

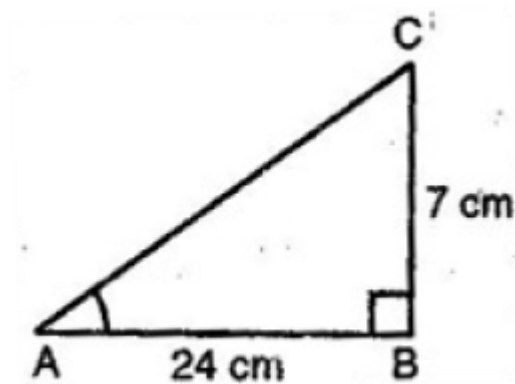
Ex 8.1 Question 1.

In $\triangle ABC$, right angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine:

- (i) $\sin A \cos A$
- (ii) $\sin C \cos C$

Answer.

Let us draw a right angled triangle ABC, right angled at B.
Using Pythagoras theorem,



Let $AC = 24k$ and $BC = 7k$

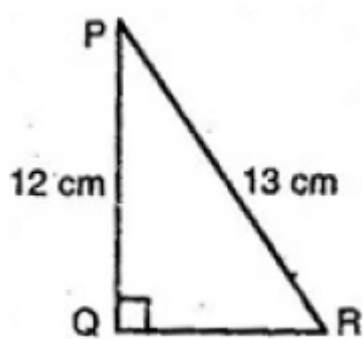
Using Pythagoras theorem,

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= (24)^2 + (7)^2 = 576 + 49 = 625 \\ \Rightarrow AC &= 25 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{(i) } \sin A &= \frac{P}{H} = \frac{BC}{AC} = \frac{7}{25}, \cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{24}{25} \\ \text{(ii) } \sin C &= \frac{P}{H} = \frac{AB}{AC} = \frac{24}{25}, \cos C = \frac{B}{H} = \frac{BC}{AC} = \frac{7}{25} \end{aligned}$$

Ex 8.1 Question 2.

In adjoining figure, find $\tan P - \cot R$:



Answer.

In triangle PQR, Using Pythagoras theorem,

$$PR^2 = PQ^2 + QR^2$$

$$\Rightarrow (13)^2 = (12)^2 + QR^2$$

$$\Rightarrow QR^2 = 169 - 144 = 25$$

$$\Rightarrow QR = 5 \text{ cm}$$

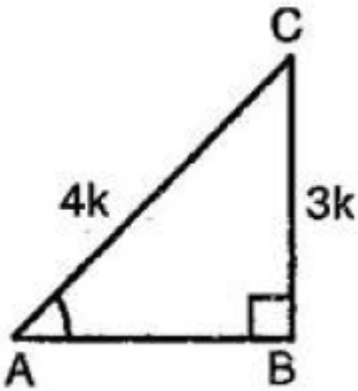
$$\therefore \tan P - \cot R = \frac{P}{B} - \frac{B}{P} = \frac{QR}{PQ} - \frac{QR}{PQ} = \frac{5}{12} - \frac{5}{12} = 0$$

Ex 8.1 Question 3.

If $\sin A = \frac{3}{4}$ calculate $\cos A$ and $\tan A$.

Answer.

Given: A triangle ABC in which $\angle B = 90^\circ$



Let $BC = 3k$ and $AC = 4k$

Then, Using Pythagoras theorem,

$$AB = \sqrt{(AC)^2 - (BC)^2} = \sqrt{(4k)^2 - (3k)^2}$$

$$= \sqrt{16k^2 - 9k^2} = k\sqrt{7}$$

$$\therefore \cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{P}{B} = \frac{BC}{AB} = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$$

Ex 8.1 Question 4.

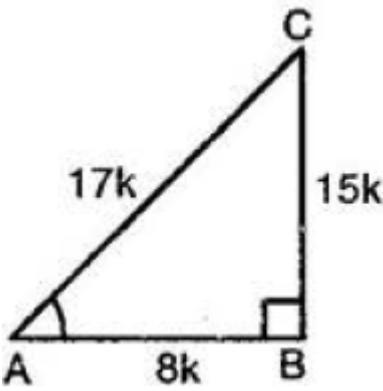
Given $15 \cot A = 8$, find $\sin A$ and $\sec A$

Answer.

Given: A triangle ABC in which $\angle B = 90^\circ$

$$15 \cot A = 8$$

$$\Rightarrow \cot A = \frac{8}{15}$$



Let $AB = 8k$ and $BC = 15k$

Then using Pythagoras theorem,

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

$$= \sqrt{(8k)^2 + (15k)^2}$$

$$= \sqrt{64k^2 + 225k^2}$$

$$= \sqrt{289k^2} = 17k$$

$$\therefore \sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

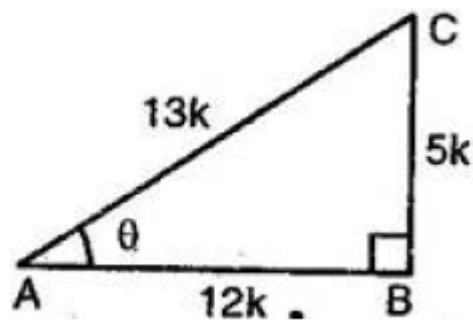
$$\sec A = \frac{H}{B} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$$

Ex 8.1 Question 5.

Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Answer.

Consider a triangle ABC in which $\angle A = \theta$ and $\angle B = 90^\circ$



Let $AB = 12k$ and $BC = 5k$

Then, using Pythagoras theorem,

$$BC = \sqrt{(AC)^2 - (AB)^2}$$

$$= \sqrt{(13k)^2 - (12k)^2}$$

$$= \sqrt{169k^2 - 144k^2}$$

$$= \sqrt{25k^2} = 5k$$

$$\therefore \sin \theta = \frac{P}{H} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos \theta = \frac{B}{H} = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan \theta = \frac{P}{B} = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot \theta = \frac{B}{P} = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$

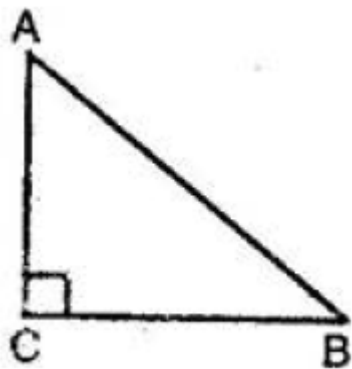
$$\operatorname{cosec} \theta = \frac{H}{P} = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$$

Ex 8.1 Question 6.

If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Answer.

In right triangle ABC,



$$\cos A = \frac{AC}{AB} \text{ and } \cos B = \frac{BC}{AB}$$

But $\cos A = \cos B$ [Given]

$$\Rightarrow \frac{AC}{AB} = \frac{BC}{AB}$$

$$\Rightarrow AC = BC$$

$$\Rightarrow \angle A = \angle B$$

[Angles opposite to equal sides are equal]

Ex 8.1 Question 7.

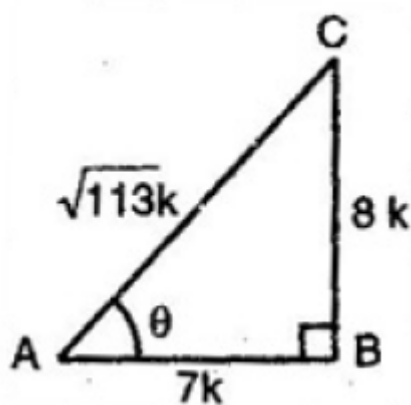
If $\cot \theta = \frac{7}{8}$, evaluate:

$$(i) \frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$$

$$(ii) \cot^2 \theta$$

Answer.

Consider a triangle ABC in which $\angle A = \theta$ and $\angle B = 90^\circ$



Let $AB = 7k$ and $BC = 8k$

Then, using Pythagoras theorem,

$$\begin{aligned} AC &= \sqrt{(BC)^2 + (AB)^2} \\ &= \sqrt{(8k)^2 + (7k)^2} \\ &= \sqrt{64k^2 + 49k^2} \\ &= \sqrt{113k^2} = \sqrt{113}k \end{aligned}$$

$$\therefore \sin \theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos \theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

$$\begin{aligned} \text{(i)} \quad \frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)} &= \frac{1-\sin^2 \theta}{1-\cos^2 \theta} \\ &= \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{113-64}{113-49} = \frac{49}{64} \end{aligned}$$

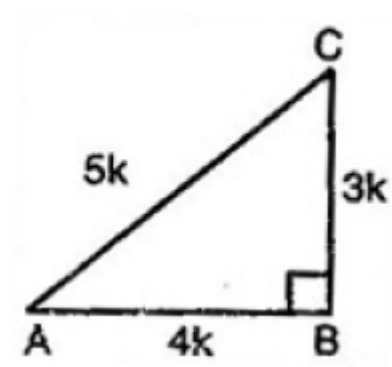
$$\text{(ii)} \quad \cot^2 \theta = \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{49/113}{64/113} = \frac{49}{64}$$

Ex 8.1 Question 8.

If $3 \cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer.

Consider a triangle ABC in which $\angle B = 90^\circ$.



And $3 \cot A = 4$

$$\Rightarrow \cot A = \frac{4}{3}$$

Let $AB = 4k$ and $BC = 3k$.

Then, using Pythagoras theorem,

$$\begin{aligned} AC &= \sqrt{(BC)^2 + (AB)^2} \\ &= \sqrt{(3k)^2 + (4k)^2} \\ &= \sqrt{16k^2 + 9k^2} \\ &= \sqrt{25k^2} = 5k \end{aligned}$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\text{And } \tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$$

$$\begin{aligned} \text{Now, L.H.S. } \frac{1-\tan^2 A}{1+\tan^2 A} &= \frac{1-\frac{9}{16}}{1+\frac{9}{16}} \\ &= \frac{16-9}{16+9} = \frac{7}{25} \end{aligned}$$

$$\begin{aligned} \text{R.H.S. } \cos^2 A - \sin^2 A &= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 \\ &= \frac{16}{25} - \frac{9}{25} = \frac{7}{25} \end{aligned}$$

$\therefore$ L.H.S. = R.H.S.

$$\therefore \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

Ex 8.1 Question 9.

In $\triangle ABC$ right angles at B, if $\tan A = \frac{1}{\sqrt{3}}$, find value of:

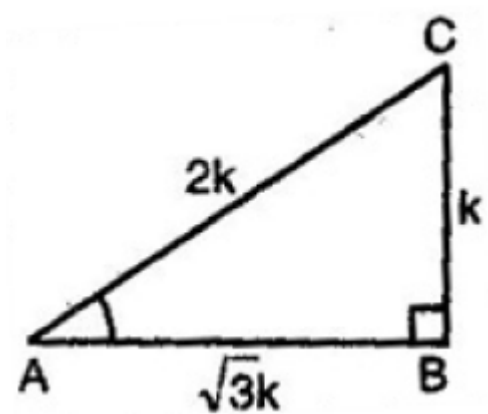
$$\text{(i)} \quad \sin A \cos C + \cos A \sin C$$

$$\text{(ii)} \quad \cos A \cos C - \sin A \sin C$$

Answer.

Consider a triangle ABC in which $\angle B = 90^\circ$.

Let $BC = k$ and $AB = \sqrt{3}k$



Then, using Pythagoras theorem,

$$\begin{aligned} AC &= \sqrt{(BC)^2 + (AB)^2} \\ &= \sqrt{(k)^2 + (\sqrt{3}k)^2} \\ &= \sqrt{k^2 + 3k^2} = \sqrt{4k^2} = 2k \\ \therefore \sin A &= \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2} \\ \cos A &= \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2} \end{aligned}$$

For $\angle C$, Base = BC, Perpendicular = AB and Hypotenuse = AC

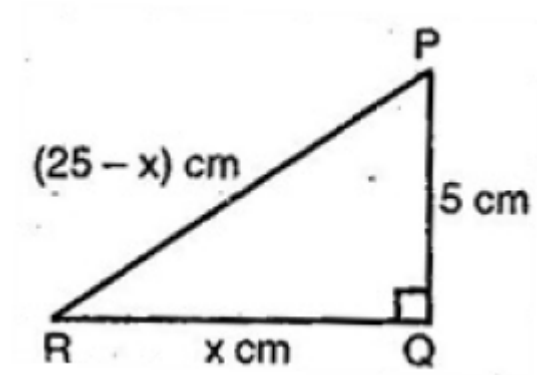
$$\begin{aligned} \therefore \sin C &= \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2} \\ \cos C &= \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2} \\ \text{(i) } \sin A \cos C + \cos A \sin C &= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \\ &= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1 \\ \text{(ii) } \cos A \cos C - \sin A \sin C &= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} \\ &= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0 \end{aligned}$$

Ex 8.1 Question 10.

In $\triangle PQR$, right angled at Q , $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

Answer.

In $\triangle PQR$, right angled at Q .



$PR + QR = 25$ cm and $PQ = 5$ cm

Let $QR = x$ cm, then $PR = (25 - x)$ cm

Using Pythagoras theorem,

$$\begin{aligned} RP^2 &= RQ^2 + QP^2 \\ \Rightarrow (25 - x)^2 &= (x)^2 + (5)^2 \\ \Rightarrow 625 - 50x + x^2 &= x^2 + 25 \\ \Rightarrow -50x &= -600 \\ \Rightarrow x &= 12 \end{aligned}$$

$\therefore RQ = 12$ cm and $RP = 25 - 12 = 13$ cm

$$\therefore \sin P = \frac{RQ}{RP} = \frac{12}{13}$$

$$\cos P = \frac{PQ}{RP} = \frac{5}{13}$$

$$\text{And } \tan P = \frac{RQ}{PQ} = \frac{12}{5}$$

Ex 8.1 Question 11.

State whether the following are true or false. Justify your answer.

- (i) The value of $\tan A$ is always less than 1 .
 (ii) $\sec A = \frac{12}{5}$ for some value of angle A .

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A .

(iv) $\cot A$ is the product of $\cot$ and A .

(v) $\sin \theta = \frac{4}{3}$ for some angle θ .

Answer.

(i) False because sides of a right triangle may have any length, so $\tan A$ may have any value.

(ii) True as $\sec A$ is always greater than 1 .

(iii) False as $\cos A$ is the abbreviation of cosine A .

(iv) False as $\cot A$ is not the product of 'cot' and A . 'cot' is separated from A has no meaning.

(v) False as $\sin \theta$ cannot be > 1 .



Exercise 8.2 (Revised) - Chapter 8 - Introduction To Trigonometry - Ncert Solutions class 10 - Maths

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Ex 8.2 Question 1.

Evaluate:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Answer.

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$



$$\begin{aligned}
& \text{(iii)} \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} \\
&= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} \\
&= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{2} \times 2(\sqrt{3}+1)} \\
&= \frac{\sqrt{3}}{\sqrt{2} \times 2(\sqrt{3}+1)} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \\
&= \frac{\sqrt{3}(\sqrt{3}-1)}{\sqrt{2} \times 2(3-1)} \\
&= \frac{\sqrt{3}(\sqrt{3}-1)}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
&= \frac{3\sqrt{2}-\sqrt{6}}{8} \\
& \quad \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ} \\
& \text{(iv)} \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}} = \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \\
&= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4} \\
&= \frac{27+16-24\sqrt{3}}{27-16} \quad [\text{SSince } (a+b)(a-b) = a^2 - b^2] \\
&= \frac{43-24\sqrt{3}}{11} \\
& \text{(v)} \frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ} \\
&= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\
&= \frac{5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{4}{4}} \\
&= \frac{15+64-12}{12} = \frac{67}{12}
\end{aligned}$$

Ex 8.2 Question 2.

Choose the correct option and justify:

(i) $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} =$

- (A) $\sin 60^\circ$
 (B) $\cos 60^\circ$
 (C) $\tan 60^\circ$
 (D) $\sin 30^\circ$

(ii) $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$

- (A) $\tan 90^\circ$
 (B) 1
 (C) $\sin 45^\circ$
 (D) 0

(iii) $\sin 2A = 2 \sin A$ is true when $A =$

- (A) 0°
 (B) 30°
 (C) 45°
 (D) 60°

(iv) $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$

- (A) $\cos 60^\circ$
 (B) $\sin 60^\circ$
 (C) $\tan 60^\circ$
 (D) None of these

Answer.

(i) (A) $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} =$

$= \frac{2 \times 1/\sqrt{3}}{1 + (1/\sqrt{3})^2}$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{3+1} = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

$$(ii) (D) \frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

(iii) (A) Since $A = 0$, then

$$\sin 2A = \sin 0^\circ = 0 \text{ and } 2 \sin A = 2 \sin 0^\circ$$

$$= 2 \times 0 = 0$$

$$\therefore \sin 2A = \sin A \text{ when } A = 0$$

$$(iv) (C) \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{3-1} = \sqrt{3} = \tan 60^\circ$$

Ex 8.2 Question 3.

If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0^\circ < B$, find A and B .

Answer.

$$\tan(A + B) = \sqrt{3}$$

$$\Rightarrow \tan(A + B) = \tan 60^\circ$$

$$\Rightarrow A + B = 60^\circ \dots\dots\dots (i)$$

$$\text{Also, } \tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan(A - B) = \tan 30^\circ$$

$$\Rightarrow A - B = 30^\circ.$$

On adding eq. (i) and (ii), we get,

$$2A = 90^\circ \Rightarrow A = 45^\circ$$

On Subtracting eq. (i) and eq. (ii), we get

$$2B = 30^\circ \Rightarrow B = 15^\circ$$

Ex 8.2 Question 4.

State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$

(ii) The value of $\sin \theta$ increases as θ increases.

(iii) The value of $\cos \theta$ increases as θ increases.

(iv) $\sin \theta = \cos \theta$ for all values of θ .

(v) $\cot A$ is not defined for $A = 0^\circ$.

Answer.

(i) False, because, let $A = 60^\circ$ and $B = 30^\circ$

$$\text{Then, } \sin(A + B) = \sin(60^\circ + 30^\circ) = \sin 90^\circ = 1$$

$$\text{And } \sin A + \sin B = \sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$$

$$\therefore \sin(A + B) \neq \sin A + \sin B$$

(ii) True, because it is clear from the table below:

θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

Therefore, it is clear, the value of $\sin \theta$ increases as θ increases.

(iii) False, because

θ	0°	30°	45°	60°	90°
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

It is clear, the value of $\cos \theta$ decreases as θ increases

(iv) False as it is only true for $\theta = 45^\circ$.

$$\Rightarrow \sin 45^\circ = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

(v) True, because $\tan 0^\circ = 0$ and $\cot 0^\circ = \frac{1}{\tan 0^\circ}$

$= \frac{1}{0}$ i.e. undefined.

Exercise 8.3 (Revised) - Chapter 8 - Introduction To Trigonometry - Ncert Solutions class 10 - Maths

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NCERT Class 10 Maths: Chapter 8 - Introduction to Trigonometry Solutions

Ex 8.3 Question 1.

Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$

Answer.

For $\sin A$,

By using identity $\operatorname{cosec}^2 A - \cot^2 A = 1$

$$\Rightarrow \operatorname{cosec} A = 1 + \cot^2 A$$

$$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A$$

$$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

For $\sec A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \tan^2 A$$

$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec^2 A = \frac{1 + \cot^2 A}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

For $\tan A$,

$$\tan A = \frac{1}{\cot A}$$

Ex 8.3 Question 2.

Write the other trigonometric ratios of A in terms of $\sec A$

Answer.

For $\sin A$ =

By using identity, $\sin^2 A + \cos^2 A = 1$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\Rightarrow \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

For $\cos A$,

$$\cos A = \frac{1}{\sec A}$$



For $\tan A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \tan^2 A = \sec^2 A - 1$$

$$\Rightarrow \tan A = \sqrt{\sec^2 A - 1}$$

For $\operatorname{cosec} A$,

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$\Rightarrow \operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

For $\cot A$,

$$\cot A = \frac{1}{\tan A}$$

$$\Rightarrow \cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

Ex 8.3 Question 3.

Evaluate:

(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

Answer.

(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$$= \frac{\sin^2 63^\circ + \sin^2 (90^\circ - 63^\circ)}{\cos^2 (90^\circ - 73^\circ) + \cos^2 73^\circ}$$

$$= \frac{\sin^2 63^\circ + \cos^2 63^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$$

$$[\because \sin(90^\circ - \theta) = \cos \theta, \cos(90^\circ - \theta) = \sin \theta]$$

$$= \frac{1}{1} = 1 \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$$= \sin 25^\circ \cdot \cos(90^\circ - 25^\circ) + \cos 25^\circ \cdot \sin(90^\circ - 25^\circ)$$

$$= \sin 25^\circ \cdot \sin 25^\circ + \cos 25^\circ \cdot \cos 25^\circ$$

$$[\because \sin(90^\circ - \theta) = \cos \theta, \cos(90^\circ - \theta) = \sin \theta]$$

$$= \sin^2 25^\circ + \cos^2 25^\circ = 1$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

Ex 8.3 Question 4.

Choose the correct option. Justify your choice:

(i) $9 \sec^2 A - 9 \tan^2 A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) =$

(A) 0

(B) 1

(C) 2

(D) none of these

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) $\sec A$

(B) $\sin A$

(C) $\operatorname{cosec} A$

(D) $\cos A$

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A} =$

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) none of these

Answer. (i) (B) $9 \sec^2 A - 9 \tan^2 A$

$$= 9 (\sec^2 A - \tan^2 A)$$

$$= 9 \times 1 = 9 \quad [\text{Since } \sec^2 \theta - \tan^2 \theta = 1]$$

(ii) (C) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$

$$= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right)$$

$$= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right)$$

$$= \frac{(\cos \theta + \sin \theta)^2 - (1)^2}{\cos \theta \cdot \sin \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta + 2 \cos \theta \sin \theta - 1}{\cos \theta \cdot \sin \theta}$$

$$= \frac{1 + 2 \cos \theta \sin \theta - 1}{\cos \theta \cdot \sin \theta}$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \frac{2 \cos \theta \sin \theta}{\cos \theta \cdot \sin \theta} = 2$$

(iii)(D) $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right) (1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right) (1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} \quad [\text{Since } (a + b)(a - b) = a^2 - b^2]$$

$$= \frac{\cos^2 A}{\cos A}$$

$$= \cos A \quad [\because 1 - \sin^2 A = \cos^2 A]$$

$$\text{(iv)(D)} \quad \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A - \tan^2 A + \tan^2 A}{\operatorname{cosec}^2 A - \cot^2 A + \cot^2 A}$$

$$= \frac{\sec^2 A}{\operatorname{cosec} A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Ex 8.3 Question 5.

Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

$$\text{(i)} \quad (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$\text{(ii)} \quad \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$\text{(iii)} \quad \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

$$\text{(iv)} \quad \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

$$\text{(v)} \quad \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A, \text{ using the identity } \operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\text{(vi)} \quad \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

$$\text{(vii)} \quad \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$$

$$\text{(viii)} \quad (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$= 7 + \tan^2 A + \cot^2 A$$

$$\text{(ix)} \quad (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$$

$$= \frac{1}{\tan A + \cot A}$$

$$\text{(x)} \quad \left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$$

Answer.

$$\text{(i) L.H.S. } (\operatorname{cosec} \theta - \cot \theta)^2$$

$$= \operatorname{cosec}^2 \theta + \cot^2 \theta - 2 \operatorname{cosec} \theta \cot \theta \quad [\text{Since } (a - b)^2 = a^2 + b^2 - 2ab]$$

$$= \frac{1}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} - 2 \times \frac{1}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta}$$

$$\begin{aligned}
&= \frac{1 + \cos^2 \theta}{\sin^2 \theta} - \frac{2 \cos \theta}{\sin^2 \theta} \\
&= \frac{1 + \cos^2 \theta - 2 \cos \theta}{\sin^2 \theta} \\
&= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} [\because a^2 + b^2 - 2ab = (a - b)^2] \\
&= \frac{(1 - \cos \theta)(1 - \cos \theta)}{1 - \cos^2 \theta} \\
&= \frac{(1 - \cos \theta)(1 - \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)} \\
&= \frac{1 - \cos \theta}{1 + \cos \theta} = \text{R.H.S.}
\end{aligned}$$

$$\begin{aligned}
\text{(ii) L.H.S. } & \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} \\
&= \frac{\cos^2 \theta + 1 + \sin^2 \theta + 2 \sin A}{(1 + \sin A) \cos A} \\
&= \frac{\cos^2 \theta + \sin^2 \theta + 1 + 2 \sin A}{(1 + \sin A) \cos A} \\
&= \frac{1 + 1 + 2 \sin A}{(1 + \sin A) \cos A} [\because \sin^2 \theta + \cos^2 \theta = 1] \\
&= \frac{2 + 2 \sin A}{(1 + \sin A) \cos A} = \frac{2(1 + \sin A)}{(1 + \sin A) \cos A}
\end{aligned}$$

$$= \frac{2}{\cos A} = 2 \sec A = \text{R.H.S}$$

$$\begin{aligned}
\text{(iii) L.H.S. } & \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} \\
&= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \\
&= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\
&= \frac{\sin \theta}{\cos \theta} \times \frac{\sin \theta}{\sin \theta - \cos \theta} + \frac{\cos \theta}{\sin \theta} \times \frac{\cos \theta}{\cos \theta - \sin \theta} \\
&= \frac{\sin^2 \theta}{\cos \theta(\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta(\cos \theta - \sin \theta)} \\
&= \frac{\sin^2 \theta}{\cos \theta(\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta(\sin \theta - \cos \theta)} \\
&= \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta(\sin \theta - \cos \theta)} \\
&= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\sin \theta \cos \theta(\sin \theta - \cos \theta)}
\end{aligned}$$

$$[\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]$$

$$\begin{aligned}
&= \frac{1 + \sin \theta \cos \theta}{\sin \theta \cos \theta} [\because \sin^2 \theta + \cos^2 \theta = 1] \\
&= \frac{1}{\sin \theta \cos \theta} + \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta}
\end{aligned}$$

$$= \frac{1}{\sin \theta \cos \theta} + 1 = 1 + \frac{1}{\sin \theta \cos \theta}$$

$$= 1 + \sec \theta \operatorname{cosec} \theta$$

$$\begin{aligned}
\text{(iv) L.H.S. } & \frac{1 + \sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\frac{\cos A + 1}{\cos A}}{\frac{1}{\cos A}} \\
&= \frac{\cos A + 1}{\cos A} \times \frac{\cos A}{1} = 1 + \cos A \\
&= 1 + \cos A \times \frac{1 - \cos A}{1 - \cos A} \\
&= \frac{1 - \cos^2 A}{1 - \cos^2 A} \quad [\text{Since } (a + b)(a - b) = a^2 - b^2] \\
&= \frac{\sin^2 A}{1 - \cos A} = \text{R.H.S.}
\end{aligned}$$

$$\text{(v) L.H.S. } \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

Dividing all terms by $\sin A$,

$$\begin{aligned}
 &= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} = \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} \\
 &= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{(1 + \cot A - \operatorname{cosec} A)} \quad [\text{Since } \operatorname{cosec}^2 \theta - \cot^2 \theta = 1] \\
 &= \frac{(\cot A + \operatorname{cosec} A) + (\cot^2 A - \operatorname{cosec}^2 A)}{(1 + \cot A - \operatorname{cosec} A)} \\
 &= \frac{(\cot A + \operatorname{cosec} A) + (\cot A + \operatorname{cosec} A)(\cot A - \operatorname{cosec} A)}{(1 + \cot A - \operatorname{cosec} A)}
 \end{aligned}$$

$$= \cot A + \operatorname{cosec} A = \text{R.H.S.}$$

$$\begin{aligned}
 \text{(vi) L.H.S. } &\sqrt{\frac{1 + \sin A}{1 - \sin A}} \\
 &= \sqrt{\frac{1 + \sin A}{1 - \sin A}} \times \sqrt{\frac{1 + \sin A}{1 + \sin A}} \\
 &= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} [\because (a + b)(a - b) = a^2 - b^2] \\
 &= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} [\because 1 - \sin^2 \theta = \cos^2 \theta] \\
 &= \frac{1 + \sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} \\
 &= \sec A + \tan A = \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii) L.H.S. } &\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} \\
 &= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)} \\
 &= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta [2 (1 - \sin^2 \theta) - 1]}
 \end{aligned}$$

$$\begin{aligned}
 &[\because 1 - \sin^2 \theta = \cos^2 \theta] \\
 &= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 - 2 \sin^2 \theta - 1)}
 \end{aligned}$$

$$= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (1 - 2 \sin^2 \theta)} = \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta = \text{R.H.S}$$

$$\begin{aligned}
 \text{(viii) L.H.S. } &(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 \\
 &= \left(\sin A + \frac{1}{\sin A} \right)^2 + \left(\cos A + \frac{1}{\cos A} \right)^2 \\
 &= \sin^2 A + \frac{1}{\sin^2 A} + 2 \cdot \sin A \cdot \frac{1}{\sin A} + \cos^2 A + \frac{1}{\cos^2 A} + 2 \cdot \cos A \cdot \frac{1}{\cos A} \\
 &= 2 + 2 + \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} \\
 &= 4 + 1 + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} \\
 &= 5 + \operatorname{cosec}^2 A + \sec^2 A \\
 &= 5 + 1 + \cot^2 A + 1 + \tan^2 A \\
 &[\because \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta, \sec^2 \theta = 1 + \tan^2 \theta] \\
 &= 7 + \tan^2 A + \cot^2 A \\
 &= \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ix) L.H.S. } &(\operatorname{cosec} A - \sin A)(\sec A - \cos A) \\
 &= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \\
 &= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \\
 &= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A \cdot \cos A \\
 &= \frac{\sin A \cdot \cos A}{\sin^2 A + \cos^2 A} [\because \sin^2 \theta + \cos^2 \theta = 1]
 \end{aligned}$$

Dividing all the terms by $\sin A \cdot \cos A$,

$$= \frac{\frac{\sin A \cdot \cos A}{\sin A \cdot \cos A}}{\frac{\sin^2 A}{\sin A \cdot \cos A} + \frac{\cos^2 A}{\sin A \cdot \cos A}}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\tan A + \cot A} = \text{R.H.S.}$$

$$(x) \text{ L.H.S. } \left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$[\because 1 + \tan^2 \theta = \sec^2 \theta, 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta]$$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \tan^2 A = \text{R.H.S.}$$

$$\text{Now, Middle side} = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \left(\frac{1 - \tan A}{1 - \frac{1}{\tan A}} \right)^2$$

$$= \left(\frac{1 - \tan A}{\frac{\tan A - 1}{\tan A}} \right)^2$$

$$= \left(\frac{1 - \tan A}{\frac{-(1 - \tan A)}{\tan A}} \right)^2 = (-\tan A)^2$$

$$= \tan^2 A = \text{R.H.S.}$$

